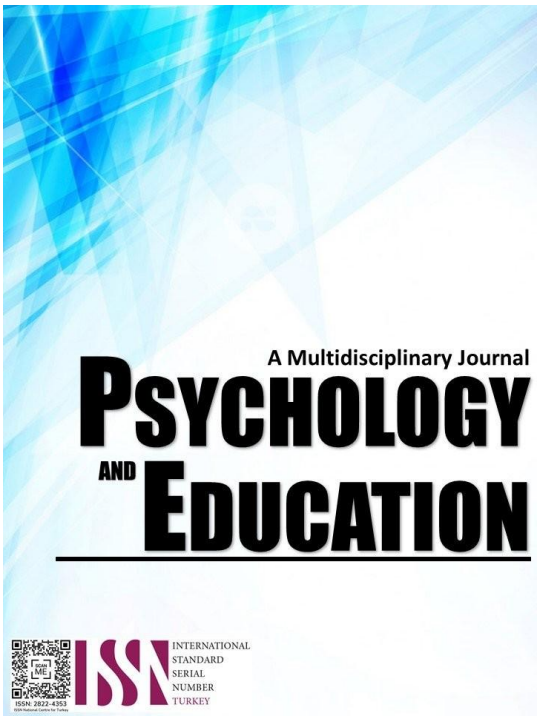
DEVELOPMENT AND VALIDATION OF QUESTIONNAIRE ON DIFFICULTIES ENCOUNTERED IN OPERATING RATIONAL NUMBERS: EXPLORATORY AND CONFIRMATORY FACTOR ANALYSES



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Research Article

Development and Validation of Questionnaire on Difficulties Encountered in Operating Rational Numbers: Exploratory and Confirmatory Factor Analyses

Kimberly A. Venus,* Ebrahim Alpe A. Simpal
For affiliations and correspondence, see the last page.

Abstract

This study developed and validated the Difficulties Encountered in Operating Rational Numbers (DEORN) questionnaire to identify learners' challenges in fractions, decimals, and negative numbers. Using a research and development design, 40 initial items derived from literature were refined through Exploratory Factor Analysis (EFA) and Confirmatory Factor Analysis (CFA). A total of 85 mathematics teachers participated in both EFA and CFA, with 20 teachers for pilot testing and 6 experts for validation. Initial sampling adequacy was low (KMO = 0.427), but after item reduction to 17 indicators, adequacy improved (KMO = 0.795), and Bartlett's test was significant (χ^2 = 475.97, df = 136, $p < 0.001$), confirming suitability for factor analysis. EFA revealed four dimensions explaining learners' difficulties: misconceptions in arithmetic operations, conceptual confusions, misapplication of natural number concepts, and errors in operations and interpretation. Factor loadings ranged from 0.459 to 0.873. Reliability analysis showed excellent internal consistency (Cronbach's α = 0.924; 95% CI: 0.883–0.953). Content validity ratings were very high across criteria, with overall means of 4.77 (usability), 4.63 (content), 4.67 (appropriateness), 4.77 (relevance), and 4.87 (acceptability). CFA results indicated good model fit (χ^2 = 11.292, df = 9, p = 0.256; CFI = 0.984; TLI = 0.974; NFI = 0.930). These findings confirm that the DEORN questionnaire is a valid and reliable diagnostic tool for identifying learners' difficulties in rational numbers.

Keywords: *difficulty encountered in operating rational numbers, exploratory and confirmatory factor analysis, validity and reliability of the developed questionnaire*

Introduction

Mathematics is widely recognized as a challenging subject, particularly in the domain of rational numbers, which encompasses fractions, decimals, and integers. Mastery of rational numbers is essential, as it serves as a foundation for advanced mathematical concepts and real-world problem-solving. However, learners frequently encounter significant difficulties in this area, often demonstrating persistent misconceptions and procedural errors. Studies have shown that these challenges are not isolated but reflect broader issues in mathematical understanding, particularly in how learners conceptualize fractions and their operations (Lamon, 2020; Kieren, 2020).

One of the pressing concerns in mathematics education is the lack of a comprehensive and validated diagnostic tool that can systematically identify learners' difficulties in operating rational numbers. Without such an instrument, teachers are limited in their ability to provide targeted and effective interventions. Traditional instructional approaches often rely on generalized strategies that fail to address individual learning needs, resulting in continued misconceptions and poor performance. The importance of developing valid and reliable instruments for educational assessment has been emphasized in research, particularly in ensuring that constructs are accurately measured and interpreted (Almanasreh et al., 2018; Boateng et al., 2018).

Learners' difficulties in rational numbers are often rooted in both procedural and conceptual misunderstandings. Many students incorrectly apply whole-number reasoning when working with fractions, leading to errors such as adding numerators and denominators directly or misinterpreting the role of denominators in comparisons. Research indicates that learners tend to treat numerators and denominators as independent values rather than components of a unified quantity (Moyo & Machaba, 2021; Reinhold et al., 2020). Additionally, misconceptions related to fraction comparison and magnitude—such as assuming that larger denominators indicate larger values—further complicate learners' understanding (Braithwaite & Siegler, 2021; González Forte et al., 2023).

Procedural errors are also prevalent in operations involving rational numbers, including addition, subtraction, multiplication, and division. Learners frequently struggle with identifying least common denominators, applying the correct operations, and handling negative signs appropriately. These difficulties are often compounded by weak foundational knowledge and limited conceptual understanding of fractions as part-whole relationships (Deringöl, 2019; Namkung et al., 2018). Furthermore, errors in interpreting mathematical signs and symbols have been documented as significant barriers to accurate computation (Bentley & Bossé, 2018; Baidoo et al., 2020).

In the Philippine educational context, the development of numeracy skills remains a national priority, as supported by policies that guarantee access to quality education. Despite these efforts, rational number operations continue to be among the least mastered competencies among learners, particularly at the secondary level. This persistent issue highlights the need for more effective diagnostic and instructional tools that can support teachers in identifying specific learning gaps. Addressing these challenges requires a shift toward evidence-based and learner-centered approaches that are grounded in both theory and empirical data (Jarrah et al., 2022; Lee & Lee, 2023).

In response to these concerns, this study aims to develop and validate the Difficulties Encountered in Operating Rational Numbers (DEORN) questionnaire. Grounded in existing literature and empirical evidence, the instrument is designed to identify the underlying factors contributing to learners' difficulties in rational number operations. By providing a systematic and reliable diagnostic tool, the study seeks to support educators in designing targeted interventions, improving instructional practices, and enhancing learners' overall mathematical understanding and performance.

Research Questions

The study aimed to develop and validate a questionnaire that could identify underlying factors that contribute to difficulties encountered by learners in operating rational numbers from available related literature and mathematics teachers in the Municipality of Malapatan. Specifically, it sought answers to the following questions:

1. What are the difficulties encountered by the learners from various literatures and studies in operating rational numbers?
2. What are the dimensions of difficulties encountered, cited from various literatures and studies in operating rational numbers from the exploratory factor analysis?
3. Based on the result of the confirmatory factor analysis, what research questionnaire can be developed?
4. What is the reliability of the developed questionnaire on the difficulty of operating rational numbers?
5. What is the level of validity on developed questionnaire on the difficulties encountered in operating rational numbers, rated by the external validators in terms of:
    5.1 usability;
    5.2 content;
    5.3 appropriateness;
    5.4 relevance; and
    5.5 acceptability?
6. What is the confirmatory factor analysis result of the Difficulties Encountered in Operating Rational Numbers (DEORN) questionnaire?

Methodology

Research Design

This study employed a Research and Development (R&D) design, which is appropriate for the systematic development, validation, and evaluation of educational instruments. R&D design integrates both empirical and iterative processes to ensure that the developed tool meets standards of validity, reliability, and practical utility (Richey, 2025). In this study, the design provided a structured framework for constructing a diagnostic questionnaire aimed at identifying learners' difficulties in operating rational numbers.

The development process was grounded in an extensive review of literature, from which recurring themes and documented misconceptions were extracted and operationalized into measurable indicators. These indicators formed the initial pool of questionnaire items, which were subjected to Exploratory Factor Analysis (EFA) to uncover latent constructs underlying the identified difficulties. The iterative refinement of the instrument involved expert validation to ensure alignment with theoretical constructs and practical relevance. The integration of Confirmatory Factor Analysis (CFA) further strengthened the instrument by validating the factor structure derived from EFA, thereby ensuring construct validity and model robustness.

Respondents

The study involved multiple groups of respondents across different phases of instrument development to ensure methodological rigor. For the Exploratory Factor Analysis, a total of 85 mathematics teachers from the Municipality of Malapatan participated. This sample size is considered adequate based on established guidelines for factor analysis, which recommend sufficient respondent-to-item ratios to ensure stable factor extraction and reliable parameter estimates.

To establish content validity, six (6) subject matter experts in mathematics education were purposively selected to evaluate the instrument. Their expertise ensured that the questionnaire items were theoretically grounded, contextually appropriate, and aligned with the intended constructs. Additionally, a pilot test was conducted with 20 mathematics teachers to assess the clarity, usability, and preliminary reliability of the instrument.

For the Confirmatory Factor Analysis, a separate sample of 85 mathematics teachers was utilized to validate the factor structure identified during the exploratory phase. The use of independent samples for EFA and CFA enhances the generalizability and robustness of the measurement model, minimizing the risk of overfitting and ensuring cross-validation of the instrument.

Instrument

The primary instrument developed in this study is the Difficulties Encountered in Operating Rational Numbers (DEORN) Questionnaire, designed to capture learners' procedural errors, conceptual misunderstandings, and misapplications of mathematical principles. The initial item pool was derived from literature-based indicators and structured using a Likert-scale format to measure

teachers' perceptions of learners' difficulties.

A five-point Likert scale, adapted from Marin and Marasigan (2003), was employed to assess the instrument's validity in terms of usability, content, appropriateness, relevance, and acceptability. The use of Likert scaling allows for nuanced responses and facilitates quantitative analysis of subjective judgments.

Exploratory and confirmatory factor analyses were conducted using Jeffrey's Amazing Statistical Program (JASP), a statistical software suitable for psychometric analysis. The instrument underwent pilot testing to evaluate its reliability and clarity prior to large-scale administration. The final version of the DEORN questionnaire reflects a psychometrically sound instrument supported by empirical validation procedures.

Procedure

The study followed a systematic, multi-phase procedure aligned with standard instrument development protocols. Initially, a comprehensive review of related literature was conducted to identify common difficulties in rational number operations. This was followed by an inductive meta-analysis, where themes were synthesized and translated into measurable indicators for item construction.

In the second phase, the preliminary questionnaire was administered to mathematics teachers to generate data for Exploratory Factor Analysis. This phase was critical in identifying the underlying dimensions of learners' difficulties and refining the item structure based on statistical and theoretical considerations.

The third phase involved conducting EFA using JASP, which resulted in the extraction of significant factors representing distinct dimensions of difficulty. Items with low factor loadings or poor communalities were removed to improve the instrument's structure.

In the fourth phase, the instrument underwent expert validation to assess its usability, content validity, appropriateness, relevance, and acceptability. Feedback from validators informed further revisions. Subsequently, pilot testing was conducted to evaluate reliability and ensure clarity of items.

The final phase involved administering the refined questionnaire to a new sample for Confirmatory Factor Analysis, which validated the factor structure and confirmed the instrument's construct validity. The finalized instrument was then prepared for implementation and dissemination.

Data Analysis

Data analysis was conducted using both qualitative and quantitative techniques to ensure comprehensive validation of the instrument. Initially, thematic analysis was employed to synthesize findings from the literature and identify recurring patterns of difficulty. These themes informed the development of questionnaire items.

For construct identification, Exploratory Factor Analysis (EFA) was performed using principal component analysis with varimax rotation to enhance interpretability. Factor retention was guided by eigenvalues greater than 1, scree plot analysis, and factor loadings of at least 0.40. Prior to EFA, data suitability was assessed using the Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy and Bartlett's Test of Sphericity. Acceptable KMO values (≥ 0.60) and significant Bartlett's test results ($p < 0.05$) confirmed the appropriateness of factor analysis.

Reliability of the instrument was evaluated using Cronbach's alpha, with values between 0.70 and 0.90 indicating acceptable to excellent internal consistency. Item analysis was conducted to identify and refine problematic items.

Content validity was assessed using the Content Validity Index (CVI), including both item-level (I-CVI) and scale-level (S-CVI) indices. Descriptive statistics, including weighted means and standard deviations, were used to summarize validators' ratings.

To confirm the factor structure, Confirmatory Factor Analysis (CFA) was conducted. Model fit was evaluated using multiple indices, including Chi-square (χ^2), Root Mean Square Error of Approximation (RMSEA ≤ 0.08), Comparative Fit Index (CFI ≥ 0.90), Tucker-Lewis Index (TLI ≥ 0.90), and Standardized Root Mean Square Residual (SRMR ≤ 0.08). These indices provided evidence of the model's adequacy and the instrument's construct validity.

Ethical Considerations

The study adhered to established ethical standards in educational research to ensure the protection and welfare of all participants. Prior to data collection, informed consent was obtained, with participants being fully informed of the study's objectives, procedures, and potential benefits.

Participation was strictly voluntary, and respondents were given the right to withdraw at any stage without penalty. Confidentiality and data privacy were rigorously maintained, with all responses treated anonymously and used solely for research purposes.

Approval to conduct the study was secured from relevant educational authorities, including Public School District Supervisors and Mathematics Coordinators. All documentation, including consent forms, was properly recorded and stored. These measures ensured

that the study complied with ethical principles of respect, beneficence, and integrity in research practice.

Results and Discussion

This section presents the results, interpretation, and analyses on the data gathered for this research.

Difficulties Encountered by the Learners in Rational Numbers from Various Literatures and Studies in Operating Rational Numbers

There are two major difficulties presented by the literature; however, there are 40 misconceptions specified. These 40 are sufficient as visualized by the researcher because of the saturation level achieved.

Table 1. *Difficulties Encountered by the learners in Operating Rational Numbers*

	Items *The learner…*	*Citation*
1.	takes the dividend always larger than the divisor when dividing both numerators and both denominators.	Abramovich and Connell (2021)
2.	divides by multiplying without taking reciprocals.	Lamon (2020)
3.	simplify by making its smaller without considering the GCF.	Moyo and Machaba (2021)
4.	simplifies the fractions by taking the larger divided by the smaller number.	Braithwaite, Tian, and Siegler (2018)
5.	simplifies the fraction by dividing through approximations.	Kieren (2020)
6.	identified the LCD but unable to multiply the numerator.	Lee (2020)
7.	adds the numerator of the first added to the denominator of the second addends and multiply the denominators.	Lee and Lee (2023)
8.	converts mixed numbers to fractions by taking the sum of the numbers.	Sowder (2020)
9.	adds the numerator by the numerator and the denominator by the denominator.	Moyo and Machaba (2021)
10.	adds the numerator of the first addend and the denominator of the second addend over the sum of the numerators, subtracting to the sum of the denominator and numerator of first addend over the sum of denominator of the first addend and the numerator of the second addend.	Braithwaite and Siegler (2021)
11.	adds the numerator of the first addend and the denominator of the second addend plus the numerator of the second addend and the denominator of the first addend.	Obersteiner and Staudinger (2018)
12.	Subtracts the denominator and numerator of the minuend minus the difference between denominator and numerator of the subtrahend.	Idris and Narayanan (2023)
13.	adds both numerators and denominators and incorrectly simplified to lowest form.	Baidoo (2019)
14.	adds both the numerator over the sum of the numerator and denominator of the first addend plus the sum of the numerator and denominator of the second addend over the sum of both denominators.	Obersteiner and Staudinger (2018)
15.	adds the numerator of the second addend and the denominator of the first addend over the sum of the numerator first addend and the denominator of the second addend plus the second addends. The result is again added to the second addends with the same pattern.	Siegler, Im, Schiller, Tian, and Braithwaite (2020).
16.	Subtracts both the numerator and the denominator.	Idris and Narayanan (2023)
17.	uses the negative sign twice when it is placed in the middle.	Bentley and Bossé (2018)
18.	divides both numerator and denominator and always divides the larger number to the smaller number.	Moyo and Machaba, (2021)
19.	divides numerator by numerator and denominator by denominator	
20.	disregards the negative sign in multiplying rational numbers	
21.	disregards the negative sign of the fraction while subtracting both numerators and both denominators	Moyo and Machaba, (2021). Kidron (2018).
22.	multiplies the numerator of the multiplicand and denominator of the multiplier over the product of the denominator of the multiplicand and the numerator of the multiplier.	Barbieri et al. (2020) Simon, Kara, and Placa (2018).
23	multiplies both the numerator and copies the denominator	Ibañez and Pentang (2021)
24.	multiplies both the numerator and divided both denominators	Ghani and Maat (2018).
25.	Subtracts the numerator of the minuend and the denominator of the subtrahend minus the numerator of the subtrahend and the denominator of the minuend.	Idris and Narayanan (2023)
26.	treats fractions as integers, they consider the numerator and the denominator as separate numbers.	Alkhateeb (2019)
27.	While expressing fractions with shapes, they cannot draw identical lines or squares. This shows that there is a problem in dividing the fraction into identical pieces in the subject of piece-whole.	Deringöl (2019)
28.	cannot understand the concepts of whole and piece.	Lamon (2020)
29.	may reflect their generalizations on natural number onto fractions and consider the numerator and the denominator as separate numbers.	Moyo and Machaba (2021)
30.	generalize their preliminary information on natural numbers in fractions when comparing fractions.	Reinhold et al. (2020)
31.	think the one with larger denominator is greater while ranking fractions with equal	González Forte et al. (2023)

	numerators.	
32.	cannot distinguish the denominator and the numerator.	Deringöl (2019)
33.	confused fractions with their knowledge on integers.	Kieren (2020)
34.	adds the numerator of the first addend and the denominator of the second addend plus the numerator of the second addend and the denominator of the first addend.	Wiese et al. (2024)
35.	had difficulty in writing and reading fractions.	Namkung, Fuchs, and Koziol (2018)
36.	concept of fractions was not fully established.	Wall (2018)
37.	think the numerator and the denominator are separate values. The main reason for this is that student use their knowledge on natural numbers when they encounter fractions.	Moyo and Machaba, (2021)
38.	think that fractions with larger denominators are greater.	Braithwaite and Siegler (2021)
39.	overlook or misunderstand the concept of negative rational numbers, leading to mistakes in operations involving negative fractions.	Baidoo, Adane, and Luneta (2020)
40.	incorrectly assume that rules for addition, subtraction, multiplication, or division of rational numbers are the same as those for whole numbers, leading to errors in calculations.	Jarrah, Wardat, and Gningue (2022)

Table 1 presents the various difficulties encountered by learners in understanding and working with fractions, along with the corresponding authors who identified these challenges. The difficulties in operating rational numbers can be categorized into key areas, including division, addition and subtraction, multiplication, and conceptual misunderstandings. Each of these reflects either misconceptions or procedural errors, as supported by existing studies.

Many learners experience difficulties in division due to a lack of understanding of the procedures or failure to apply the rules correctly. One common issue is the misordering of the dividend and divisor, where learners assume that the dividend must always be larger than the divisor (Abramovich & Connell, 2021). Another difficulty is the failure to apply reciprocals, as learners often divide fractions by directly multiplying them without inverting the divisor (Lamon, 2020). Additionally, learners tend to rely on approximations when simplifying fractions, using estimates rather than exact values (Kieren, 2020). Misalignment between numerators and denominators is also evident, as learners treat them as independent quantities rather than components of a single value (Moyo & Machaba, 2021). Furthermore, incorrect division logic arises when learners apply natural number reasoning to fractions, resulting in flawed computational processes (Reinhold et al., 2020).

Errors in addition and subtraction of fractions are often attributed to improper handling of numerators and denominators. One common issue is the incorrect use of the least common denominator (LCD), where learners identify the LCD but fail to adjust the numerators accordingly (Lee & Boyadzhiev, 2020). Another frequent error involves adding numerators and denominators directly instead of forming equivalent fractions (Moyo & Machaba, 2021; Baidoo, 2019). Learners also exhibit complex procedural errors, such as misapplying addition and subtraction rules by interchanging numerators and denominators across fractions (Braithwaite & Siegler, 2021). Moreover, difficulties in handling negative signs during subtraction often lead to incorrect results (Idris & Narayanan, 2023).

In multiplication, errors generally stem from incorrect application of rules. Learners often misalign numerators and denominators, leading to confusion about which elements should be multiplied (Simon et al., 2018). Additionally, the neglect or improper handling of negative signs contributes to computational errors (Kidron, 2018). Another common issue is treating fractions as whole numbers, where learners multiply numerators and denominators independently without considering their relational meaning (Moyo & Machaba, 2021).

These procedural difficulties reflect deeper conceptual challenges in understanding rational numbers. Learners often struggle with the part-whole relationship inherent in fractions, indicating confusion between whole numbers and fractional representations (Lamon, 2020; Deringöl, 2019). Difficulties in ranking and comparing fractions are also prevalent, as learners tend to assume that fractions with larger denominators are greater (González-Forte et al., 2023). Furthermore, misunderstandings related to negative fractions lead to significant operational errors (Baidoo et al., 2020). The overgeneralization of whole-number rules to fractions further exacerbates these issues, as learners incorrectly apply familiar rules in inappropriate contexts (Jarrah et al., 2022).

Overall, the literature highlights that learners' difficulties in rational numbers stem from a combination of procedural errors, misconceptions, and insufficient foundational understanding. These challenges are interconnected and often rooted in broader issues of mathematical literacy, particularly the overgeneralization of natural number concepts and limited exposure to meaningful fraction representations.

Moyo and Machaba (2021) consistently emphasize the difficulties learners face when applying whole-number reasoning to fractions. Similarly, Lee and Boyadzhiev (2020) highlight procedural errors related to the use of the LCD and adjustment of numerators, while Reinhold et al. (2020) stress the importance of conceptual clarity in fraction comparison.

Several studies have documented misconceptions in arithmetic operations involving fractions. Wiese et al. (2024), Ghani and Maat (2018), Braithwaite and Siegler (2021), Idris and Narayanan (2023), Baidoo (2019), Braithwaite et al. (2021), Simon et al. (2018), Obersteiner and Staudinger (2018), Kieren (2020), and Abramovich and Connell (2021) reported that learners frequently demonstrate misunderstandings in fundamental operations, such as incorrect addition, improper multiplication without considering denominators,

and inaccurate simplification of fractions.

In terms of sign interpretation, Bentley and Bossé (2018), Barbieri et al. (2020), Alkhateeb (2019), and Baidoo et al. (2019) found that learners struggle with the correct application of mathematical signs, particularly negative signs. These studies observed that learners often disregard or misuse negative signs, leading to errors in fraction operations.

Furthermore, Moyo and Machaba (2021), Kieren (2020), Namkung et al. (2018), Wall (2018), and Reinhold et al. (2020) identified persistent conceptual confusion between fractions and whole numbers or integers. Learners tend to conflate these numerical systems, resulting in flawed reasoning and incorrect operations.

Additional studies by Reinhold et al. (2020), González-Forte et al. (2019), Deringöl (2019), and Lee and Lee (2023) revealed that learners face challenges in comparing fractions, representing them visually, and understanding their relative magnitudes. These difficulties are often linked to the inappropriate generalization of natural number properties, particularly when comparing fractions based on denominator size.

Moreover, Moyo and Machaba (2021), Braithwaite et al. (2021), Deringöl (2019), and Idris and Narayanan (2023) consistently reported that learners struggle to distinguish between numerators and denominators. Learners frequently treat these components as separate and unrelated values, indicating a fundamental misunderstanding of fraction structure.

Finally, studies by Wall (2018), Namkung et al. (2019), and Deringöl (2019) demonstrated that many learners' errors originate from weak foundational knowledge of fractions. These learners exhibit incomplete conceptual development, which manifests in difficulties in reading, writing, and interpreting fractions accurately.

Dimensions of Difficulties Encountered Cited from Various Literatures and Studies in Operating Rational Numbers from the Exploratory Factor Analysis

Before conducting EFA, the Kaiser-Meyer-Olkin (KMO) of sampling adequacy was examined for all 40 items. The KMO measure was not substantial (KMO=0.427), indicating that the overall measure of sampling adequacy for the dataset is quite low. This suggests that the data may not be suitable for factor analysis if the low MSA (measure of sampling adequacy) is not removed. Hence, the following items with low MSA were removed. From the 40 items, the following items were considered to be retained. Items 1, 5, 6, 10, 12, 14, 16, 17, 2, 24, 25, 29, 30, 32, 33, 36, and 37.

Table 2. *Kaiser-Meyer-Olkin Test for Sampling Adequacy*

Items	*MSA*
1. The learner takes the dividend always larger than the divisor when dividing both numerators and both denominators.	0.760
5. The learner simplifies by dividing through approximations.	0.859
6. The learner identified the LCD but unable to multiply the numerator.	0.746
10. The learner adds the numerator of the first addend and the denominator of the second addend over the sum of the numerators, subtracting the sum of the denominators and numerators of the first addend over the sum of the denominator of the first addend and the numerator of the second addend.	0.808
12. The learner subtracts the denominator and numerator of the minuend minus the difference between the denominator and the numerator of the subtrahend.	0.835
14. The learner adds both the numerator over the sum of the numerator and denominator of the first addend plus the sum of the numerator and denominator of the second addend over the sum of both denominators.	0.867
16. The learner subtracts both the numerator and the denominator.	0.703
17. The learner uses the negative sign twice when it is placed in the middle.	0.786
21. The learner disregards the negative sign of the fraction while subtracting both numerators and both denominators.	0.803
24. The learner multiplies both the numerator and divided both denominators.	0.782
25. The learner subtracts the numerator of the minuend and the denominator of the subtrahend minus the numerator of the subtrahend and the denominator of the minuend.	0.802
29. Learners may reflect their generalizations on natural number onto fractions and consider the numerator and the denominator as separate numbers.	0.792
30. While comparing fractions, learners generalize their preliminary information on natural numbers in fractions.	0.818
32. The learners have difficulty in distinguishing the denominator and the numerator.	0.759
33. The learners 'confused fractions with their knowledge on integers.	0.802
36. The concept of fractions was not fully established.	0.846
37. Learners think the numerator and the denominator are separate values. The main reason for this is that student use their knowledge on natural numbers when they encounter fractions.	0.723
Overall MSA	0.795

Table 2 shows that the KMO of sampling adequacy was reexamined for the 17 items. A KMO value of 0.795 suggests that the sample is adequate for factor analysis. This entails that the factors extracted will likely represent the underlying structure of the data accurately.

Before performing factor analysis, it is essential to verify whether the data are suitable for this type of analysis. One common measure is Bartlett's test of sphericity, which examines whether the correlation matrix significantly differs from an identity matrix. A

significant result indicates that variables are sufficiently correlated to justify factor analysis.

Table 3. *Bartlett's Test of Sphericity*

Test Statistic	*Degrees of Freedom*	χ^2	*p-value*	*Remarks*
Bartlett's Test	136	475.97	0.001	Significant

In Table 3, Bartlett's test was significant (X2 = 475.97, df=136, p <.001), indicating that the correlation matrix was significantly different from an identity matrix. A significant Bartlett's test indicates that the correlation matrix is not an identity matrix, supporting the suitability of the data for factor analysis. The result showed that performing EFA is allowed. Thus, varimax rotation was undertaken.

Table 4 contains factor loadings and uniqueness values for 17 cleaned items in a factor analysis. Factor loadings indicate the strength and direction of the relationship between each item and the underlying extracted factors. Each item is associated with one or more factors, and higher loadings indicate a stronger relationship with that factor, providing insight into how the items contribute to the overall factor structure. Uniqueness values reflect the proportion of variance in each item not explained by the factors, helping to assess the adequacy of the factor model. Examining both factor loadings and uniqueness values is essential for evaluating the adequacy of the factor structure, ensuring that the retained factors meaningfully represent the underlying dimensions measured by the questionnaire.

Table 4. *Rotated Component Matrix on Factor Loadings from the 17 Cleaned Items*

Indicators	*Factor 1*	*Factor 2*	*Factor 3*	*Factor 4*
25. The learner subtracts the numerator of the minuend and the denominator of the subtrahend minus the numerator of the subtrahend and the denominator of the minuend.	0.873			
12. The learner subtracts the denominator and numerator of the minuend minus the difference between the denominator and the numerator of the subtrahend.	0.853			
14. The learner adds both the numerator over the sum of the numerator and denominator of the first addend, plus the sum of the numerator and denominator of the second addend over the sum of both denominators.	0.701			
10. The learner adds the numerator of the first addend and the denominator of the second addend over the sum of the numerators, subtracting the sum of the denominators and numerators of the first addend over the sum of the denominator of the first addend and the numerator of the second addend.	0.645			
24. The learner multiplies both the numerator and divided both denominators.	0.640			
16. The learner subtracts both the numerator and the denominator.		0.820		
37. Learners think the numerator and the denominator are separate values. The main reason for this is that the student use their knowledge on natural numbers when they encounter fractions.		0.816		
33. The learners 'confused fractions with their knowledge on integers.		0.638		
36. The concept of fractions was not fully established.		0.626		
30. While comparing fractions, learners generalize their preliminary information on natural numbers in fractions.			0.828	
29. Learners may reflect their generalizations on natural number onto fractions and consider the numerator and the denominator as separate numbers.			0.754	
32. The learners have difficulty in distinguishing the denominator and the numerator.			0.565	
1. The learner takes the dividend always larger than the divisor when dividing both numerators and both denominators.				0.697
5. The learner simplifies by dividing through approximations.				0.468
21. The learner disregards the negative sign of the fraction while subtracting both numerators and both denominators.				0.594
17. The learner uses the negative sign twice when it is placed in the middle.				0.459

Note: Applied rotation method is varimax.

Based on the result, the first dimension is Factor 1, which contains the items 25, 12, 14, 10, and 24. Item 25, the learner subtracts the numerator of the minuend, and the denominator of the subtrahend, minus the numerator of the subtrahend and the denominator of the minuend, and item 12, the learner subtracts the denominator and numerator of the minuend minus the difference between the denominator and the numerator of the subtrahend, has the highest loading. Items 25 and 12 have the higher loadings (r=0.873; 0.853) representing specific behaviors related to subtraction of fractions. The uniqueness values for these items are relatively low, indicating that a significant portion of the variance in these items is explained by Factor 1. Thus, the first dimension is difficulty in subtracting fractions.

Additionally, the second dimension is Factor 2, which includes the items 16, 37, 33, and 36. Item 16, the learner subtracts both the numerator and the denominator, and 37 learners think the numerator and the denominator are separate values. The main reason for this is that learners use their knowledge of natural numbers when they encounter fractions, which have the highest loadings (r=0.820; 0.816), which appear to capture misunderstandings or misconceptions related to fractions, particularly regarding the interpretation of

numerators and denominators. The uniqueness values for these items are relatively low, indicating that a significant portion of the variance in these items is explained by Factor 2. Thus, the second dimension is the misconceptions of numerators and denominators.

Moreover, the third dimension is Factor 3, which includes items 30, 29, and 32. Based on the factor loadings and uniqueness values provided for Factor 3, it seems to capture behaviors and misconceptions related to learners' understanding and comparison of fractions. Item 30: While comparing fractions, learners generalize their preliminary information on natural numbers, and fractions have the highest factor loading. It suggests that learners tend to apply their understanding of natural numbers to fractions when comparing them, indicating a misunderstanding of the unique properties of fractions. On item 29, Learners may reflect their generalizations on natural number onto fractions and consider the numerator and the denominator as separate numbers, which further supports the idea that learners extend their understanding of whole numbers to fractions, treating numerators and denominators as separate entities. While in item 32, the learners have difficulty in distinguishing the denominator and the numerator, it reveals that a common difficulty learners face is in correctly identifying and distinguishing between numerators and denominators in fractions. Hence the dimension 3 is linked to misconceptions related to learners' conceptualization of fractions, particularly their tendency to generalize from whole numbers and struggle with distinguishing numerators and denominators.

Lastly, the fourth dimension captures the items 5, 21, 1, 6, and 17. From the factor loadings, these items are related to specific misconceptions or errors in how learners approach mathematical operations, particularly involving fractions. Items 5, 1, and 6 reflect misconceptions in how learners execute mathematical operations, such as division and simplification, particularly when dealing with fractions. While items 21 and 17 show errors related to the interpretation or manipulation of signs, particularly negative signs, in fraction operations. Hence, dimension 4 is about difficulties in correctly interpreting and applying signs, such as negative signs, in fraction operations.

Table 5 shows the items, indicators, factors, and themes for the questionnaire. Items 25, 12, 14, 10, and 24 revealed learners' struggles with arithmetic operations involving fractions, including subtraction and multiplication. These misconceptions involve applying incorrect rules, such as subtracting across both numerators and denominators or performing operations like multiplication incorrectly in dividing denominators rather than multiplying. This dimension provides the importance of addressing procedural misconceptions, as learners frequently attempt operations without a proper understanding of mathematical principles.

Table 5. *Dimensions of Difficulties Encountered Cited from Various Literatures and Studies in Operating Rational Numbers*

Item No.	*Indicators*	*Factor No.*	*Themes or Dimensions*
25, 12, 14, 10, and 24	The learner subtracts the numerator of the minuend and the denominator of the subtrahend minus the numerator of the subtrahend and the denominator of the minuend in subtraction of a fraction. The learner subtracts the denominator and numerator of the minuend minus the difference between the denominator and the numerator of the subtrahend. The learner adds both the numerator over the sum of the numerator and denominator of the first addend plus the sum of the numerator and denominator of the second addend over the sum of both denominators. The learner adds the numerator of the first addend and the denominator of the second addend over the sum of the numerators, subtracting the sum of the denominators and numerators of the first addend over the sum of the denominator of the first addend and the numerator of the second addend. The learner multiplies both the numerator and divided both denominators. The learner subtracts both the numerator and the denominator.	1	Misconceptions in performing arithmetic operations with fractions
16, 37, 33, and 36	Learners think the numerator and the denominator are separate values. The main reason for this is that students use their knowledge on natural numbers when they encounter fractions. The learners confused fractions with their knowledge on integers. The learners' concept of fractions was not fully established.	2	Conceptual Confusions in Fraction Understanding
30, 29, and 32	While comparing fractions, learners generalize their preliminary information on natural numbers in fractions. Learners may reflect their generalizations on natural number onto fractions and consider the numerator and the denominator as separate numbers. The learners have difficulty in distinguishing the denominator and the numerator.	3	Misapplication of Natural Number Concepts to Fractions
5, 21, 1, 6, and 17	The learner takes the dividend always larger than the divisor when dividing both numerators and both denominators. The learner disregards the negative sign of the fraction while subtracting both numerators and both denominators. The learner identified the LCD but unable to multiply the numerator. The learner uses the negative sign twice when it is placed in the middle.	4	Errors in Fraction Operations and Interpretation

Items 16, 37, 33, and 36 reveal challenges related to learners' conceptual grasp of fractions. Errors include viewing numerators and

denominators as independent entities or conflating fractions with integers. These issues started from a lack of foundational understanding, as learners fail to internalize fractions as parts of a whole or as numbers representing proportional relationships. Without resolving these conceptual confusions, procedural instruction is difficult to yield meaningful improvements in learners' performance.

Items 30, 29, and 32 demonstrate how learners inappropriately generalize properties of natural numbers when working with fractions. For instance, they may assume that fractions with larger numerators or denominators are automatically greater, reflecting a misunderstanding of fraction comparison. Additionally, learners often struggle to distinguish between the numerator and denominator. This dimension echoes the need for instructional strategies that emphasize the differences between natural numbers and fractions, helping learners to reframe their existing number sense.

Lastly, items 5, 21, 1, 6, and 17 pertain to operational and interpretational errors, such as failing to account for the reciprocal in division, disregarding the negative sign in operations, or oversimplifying fractions. These errors may arise from incomplete procedural knowledge or lapses in attention to detail, such as recognizing signs or following the order of operations. This theme suggests a need for both practice and conceptual reinforcement to ensure accurate execution of fraction operations.

Developed Questionnaire Based on the Result of the Confirmatory Factor Analysis

The Difficulties Encountered in Operating Rational Numbers questionnaire (DEORN questionnaire) was developed to identify the specific challenges learners face when performing operations with rational numbers, including addition, subtraction, multiplication, and division. The questionnaire provides insight into common misconceptions, procedural errors, and conceptual misunderstandings that may hinder learners' ability to work accurately with fractions, decimals, and negative numbers.

Based on the indicators, a suitable Dimension 1 could be Misconceptions in Performing Arithmetic Operations with Fractions. This theme includes various errors and misunderstandings that learners may encounter when performing operations with fractions. In the new Indicator 1, the learner subtracts components of fractions in a manner inconsistent with proper subtraction. This indicator shows a misconception or error in subtracting fractions, indicating a lack of understanding of the subtraction operation.

Similar to Indicator 1, this indicator also reflects a misunderstanding of fraction subtraction, suggesting a broader trend of misconceptions in this area. Indicator 3 states that the learner adds fractions using a non-standard approach that involves adding numerators and denominators separately. This indicator demonstrates a misconception in fraction addition, indicating a lack of understanding of how fractions should be combined. In Indicator 4, the learner employs a substandard method for adding fractions, which involves complex manipulations of numerators and denominators. Like the previous indicators, this suggests a misunderstanding of fraction addition and reveals a tendency to apply incorrect procedures. In indicator 5, the learner attempts to multiply fractions by incorrectly manipulating both numerators and denominators. This indicator shows a misconception in fraction multiplication, indicating a lack of understanding of the multiplication operation with fractions.

The dimension 2 theme that emerges is Conceptual Confusions in Fraction Understanding. This theme revealed the challenges learners face in grasping the fundamental concepts of fractions and their relationship to whole numbers and integers. Indicator 6 states that the learner incorrectly treats the numerator and denominator of fractions as separate entities during subtraction. This indicator reveals a fundamental misunderstanding of the relationship between the numerator and denominator in fractions, indicating a broader conceptual confusion. Indicator 7: learners perceive the numerator and denominator of fractions as independent values, influenced by their understanding of natural numbers. This indicator emphasizes the influence of prior knowledge of natural numbers on learners' perception of fractions, suggesting a lack of integration between these conceptual frameworks. Indicator 8: The learners exhibit confusion between fractions and integers, possibly conflating the properties and operations of the two. This indicator emphasizes a blurring of boundaries between fractions and integers, indicating a broader conceptual confusion regarding the distinct characteristics of these number systems. Lastly, in Indicator 9, the concept of fractions is not fully established among learners. This indicator directly addresses the incomplete understanding of fractions, indicating a need for further conceptual development and clarification.

Dimension 3 commonality that emerges is the misapplication of Natural Number Concepts to Fractions. This theme revolves around the tendency of learners to apply their understanding of natural numbers to fractions, leading to misconceptions and difficulties in fraction comparison and the identification of the numerator and denominator. In Indicator 10, Learners generalize their knowledge of natural numbers when comparing fractions. This indicates a tendency to apply natural number concepts, which may not be appropriate for fractions, leading to inaccuracies or misconceptions in fraction comparison. While Indicator 11, learners reflect their understanding of natural numbers onto fractions, treating the numerator and denominator as separate entities. This emphasizes a misapplication of natural number concepts to fractions, where learners mistakenly view the numerator and denominator as independent values, disregarding their interdependent relationship in fractions. Moreover, in Indicator 12, learners struggle to differentiate between the numerator and the denominator in fractions. This suggests a difficulty in understanding the unique characteristics and roles of numerator and denominator in fractions, potentially stemming from a tendency to apply natural number concepts where they may not be applicable.

Dimension 4 is Errors in Fraction Operations and Interpretation. This factor emphasizes various mistakes and misunderstandings that

learners encounter when performing arithmetic operations with fractions and interpreting their properties. Indicator 13, the learner simplifies fractions by dividing through approximations. This indicates an error in fraction simplification, where learners resort to approximations rather than applying proper simplification techniques.

Additionally, Indicator 14, the learner disregards negative signs when subtracting fractions. This entails a misinterpretation of the role of negative signs in fractions, leading to incorrect subtraction results. Indicator 15: The learner consistently chooses the dividend to be larger than the divisor when dividing fractions. This suggests a systematic error in fraction division, where learners consistently prioritize the dividend over the divisor, resulting in inaccurate division outcomes. Indicator 16, the learner correctly identifies the least common denominator (LCD) but struggles to multiply the numerator. This indicates a specific difficulty in executing fraction multiplication, despite understanding the concept of the LCD.

Lastly, Indicator 17, the learner incorrectly uses the negative sign twice when placed in the middle of a fraction. This reveals a misunderstanding of how negative signs should be placed and interpreted in fractions, leading to errors in fraction representation.

Reliability of the Developed DEORN-Questionnaire

The reliability of the developed Difficulties Encountered in Operating Rational Numbers (DEORN) questionnaire was assessed to determine the consistency and stability of the instrument. Reliability analysis is essential in evaluating whether the questionnaire produces dependable and consistent results across items. Establishing the reliability of the DEORN questionnaire ensures that it is a credible tool for identifying learners' difficulties in operating rational numbers.

Table 6. *Reliability of the Developed DEORN-Questionnaire*

Estimate	*Cronbach's α*
Point estimate	0.924
95% CI lower bound	0.883
95% CI upper bound	0.953

Table 6 displays the reliability analysis of the Difficulties Encountered in Operating Rational Numbers (DEORN-questionnaire) using Cronbach's alpha, which is a measure of internal consistency or reliability of a set of scale or test items. The Cronbach's alpha is 0.924. This represents the estimated reliability coefficient of the questionnaire. A Cronbach's alpha of 0.924 indicates excellent internal consistency. This means that the items in the questionnaire are highly correlated and consistently measure the same underlying construct. Typically, a Cronbach's alpha value above 0.9 is considered excellent, between 0.8 and 0.9 is considered good, and between 0.7 and 0.8 is acceptable. Therefore, a value of 0.924 is very strong, suggesting that the questionnaire is reliable. The high point estimate and the narrow confidence interval range (0.883 to 0.953) suggest that the DEORN-questionnaire is very reliable.

Level of Validity on Developed Questionnaire on the Difficulties Encountered in Operating Rational Numbers

The level of validity of the developed questionnaire on the Difficulties Encountered in Operating Rational Numbers was evaluated in terms of usability, content, appropriateness, relevance, and acceptability. Establishing validity is essential to ensure that the instrument accurately measures the intended constructs and effectively captures the difficulties experienced by learners. Each criterion provides a comprehensive assessment of the questionnaire's quality, determining whether the items are suitable, meaningful, and aligned with the objectives of the study. This evaluation supports the overall credibility and effectiveness of the developed instrument.

The level of validity of the developed questionnaire in terms of usability was evaluated to determine how easily and effectively the instrument can be utilized by respondents. Usability focuses on the clarity of instructions, simplicity of format, and overall ease of understanding of the questionnaire items. It also considers the logical organization of items, readability of language, and the time required to complete the questionnaire. Assessing usability ensures that respondents can complete the instrument without difficulty, thereby enhancing the accuracy and reliability of the data collected. Furthermore, a highly usable questionnaire increases respondent engagement and reduces the likelihood of incomplete or inconsistent responses.

Table 7. *Level of Validity on the Developed Questionnaire in Terms of Usability*

Items *The questionnaire…*	*Mean*	*Description*
1. evaluates the overall layout and structure of the questionnaire for user-friendliness.	5.00	Very High
2. assesses the ease with which participants can navigate through the questionnaire.	4.33	Very High
3. considers the time required for participants to complete it.	5.00	Very High
4. evaluates the readability, including font size and style.	4.83	Very High
5. obtains an overall impression of the usability.	4.67	Very High
Overall Mean	4.77	Very High

Table 7 presents the level of validity of the developed questionnaire in terms of usability, as evaluated by a group of external validators. The table outlines five specific criteria used to assess the usability of the questionnaire, along with their corresponding weighted means and verbal descriptions. The results indicate that the questionnaire was highly rated in terms of its usability, as

demonstrated by the Strongly Agree responses. The overall design of the questionnaire has a rating of 5.00, which falls under the Strongly Agree category, indicating that validators strongly agree that the questionnaire is well-structured and easy for participants to follow. This suggests that the questionnaire is designed in a way that minimizes confusion and facilitates smooth navigation, a key factor in enhancing user experience.

The weighted mean of 4.33 still reflects a strong agreement with the statement, although slightly lower than the previous item. This suggests that while the questionnaire is generally easy to navigate, there may be small areas that could be further refined for even smoother flow. A perfect weighted mean of 5.00 signifies that those validators strongly agree that the time allotted for the questionnaire is appropriate, neither too long nor too short. This indicates that the questionnaire has been well-calibrated to ensure participants can complete it without feeling rushed or overwhelmed, contributing to a better overall experience. A high rating of 4.83 suggests that validators strongly agree that the text is clear and easy to read. The use of appropriate font size and style ensures that participants do not experience eye strain or difficulty in understanding the questions, contributing to a more accessible and user-friendly questionnaire. With a weighted mean of 4.67, this indicates that the external validators strongly agree that the questionnaire is highly usable overall. The strong agreement reflects a general satisfaction with the usability features, suggesting that the questionnaire is effective in achieving its goal of user-friendliness.

The overall mean of 4.77 further reinforces the positive feedback, falling in the Strongly Agree range. This indicates that, on average, the external validators rated the questionnaire's usability very highly across all aspects, signifying that the questionnaire is likely to be well-received by participants in terms of ease of use, clarity, and efficiency. Consistent with the study of Hoto (2024), the initial face validity study of a tool indicates high agreement percentages signified their alignment, they are efficient, user-friendly, safe, and well-received, and thus, it brings enhanced clarity and efficacy.

Moreover, the high level of agreement observed in the initial face validity suggests that the instrument is not only aligned with its intended purpose but also meets the expectations of its intended users. Such an agreement reflects that the questionnaire items are clearly stated, appropriately structured, and free from ambiguity, allowing respondents to interpret them consistently. This alignment further supports the practicality of the tool in real research settings, as it promotes ease of administration and facilitates accurate data collection.

The level of validity of the developed questionnaire in terms of content was assessed to determine the extent to which the items adequately represent the domain being measured. Content validity ensures that the questionnaire covers all relevant aspects of the construct and reflects the intended objectives of the study. Furthermore, expert evaluation plays a crucial role in establishing content validity, as specialists review the items to ensure their accuracy, relevance, and alignment with the theoretical framework.

Table 8. *Level of Validity on Developed Questionnaire in Terms of Content*

Items *The questionnaire…*	*Mean*	*Description*
1. applies all the defined themes found in existing literature and studies on the rational number operations.	5.00	Very High
2. ensures that each indicator is simple and easy to understand.	4.33	Very High
3. assesses the alignment of the indicators to the idea of the existing literature and related studies.	4.83	Very High
4. considers whether each indicator grasps the required knowledge to better understand learners' difficulties.	4.17	High
5. obtains an overall impression of the content	4.83	Very High
Overall Mean	4.63	Very High

Table 8 provides an overview of the level of validity of the developed questionnaire in terms of content, as assessed by external validators. This table presents five specific criteria used to evaluate how well the questionnaire's content aligns with established literature on rational number operations.

A perfect weighted mean of 5.00 indicates that validators strongly agree that the questionnaire accurately reflects the relevant themes from the literature. This suggests that the content of the questionnaire is comprehensive and grounded in well-established theories and studies, ensuring it is scientifically sound. With a weighted mean of 4.33, the validators strongly agree that the indicators are generally easy to understand, although this score is slightly lower than the first item. This indicates that, while most participants will likely find the questions clear, there may be a few instances where simplification or clarification could improve comprehension for a wider audience.

A weighted mean of 4.83 indicates strong agreement, suggesting that the questionnaire's indicators are highly consistent with the theoretical frameworks and previous research findings on rational number operations. With a weighted mean of 4.17, the item falls within the Agree category, suggesting that while most validators are satisfied with how the indicators address the learners' difficulties, there may be some room for further improvement. The weighted mean of 4.83 reflects that the validator strongly agrees that the content of the questionnaire is well-crafted and relevant. This high score indicates a positive assessment of the overall quality and relevance of the questionnaire's content, reinforcing the notion that the instrument is grounded in valid, research-based themes and concepts.

The overall mean of 4.63 suggests that, overall, the content validity of the questionnaire is strong. The average score falls within the

Strongly Agree range, indicating that the content of the questionnaire is highly regarded by the external validators. It is clear from this table that the questionnaire is well-aligned with existing literature and effectively captures the necessary themes related to rational number operations.

Consistent with the study of Almanasreh et al. (2018), content validation processes and content validity indices are essential factors in the instrument development process, and should be treated and reported as important as other types of construct validation. Content validity deserves a rigorous assessment process as the obtained information from this process is invaluable for the quality of the newly developed instrument, questionnaire showed acceptable to good psychometric properties.

Furthermore, a rigorous content validation process strengthens the overall integrity of the instrument by ensuring that each item is both relevant and representative of the construct being measured. The use of content validity indices provides quantitative evidence to support expert judgments, allowing for more objective evaluation of the questionnaire's adequacy. This process also helps identify items that may require revision, elimination, or refinement, thereby improving the coherence and focus of the instrument.

The level of validity of the developed questionnaire in terms of appropriateness was evaluated to determine whether the items are suitable and relevant for the target respondents and the objectives of the study. Appropriateness assesses whether the content, language, and structure of the questionnaire align with the participants' knowledge, experience, and context.

Table 9. *Level of Validity on the Developed Questionnaire in Terms of Appropriateness*

Items *The questionnaire…*	*Mean*	*Description*
1. assesses the alignment of the indicators with the overall research objectives.	4.83	Very High
2. evaluates the language, content, and tone of the indicators suited for the target population.	4.50	Very High
3. Consider all indicators that are culturally sensitive and suitable for diverse groups.	4.83	Very High
4. ensures all the indicators are ethically sound and considers participant well-being.	4.67	Very High
5. obtains an overall impression of the appropriateness.	4.50	Very High
Overall Mean	4.67	Very High

Table 9 presents the level of validity of the developed questionnaire in terms of appropriateness, as assessed by external validators. This table includes five specific criteria that examine how well the questionnaire aligns with the research objectives, its suitability for the target population, and its cultural and ethical considerations. A weighted mean of 4.83 suggests that the validators strongly agree that the questionnaire's indicators are closely aligned with the research objectives. This high score indicates that the questionnaire effectively addresses the intended research questions and is highly relevant. With a weighted mean of 4.50, validators strongly agree that the questionnaire's language and content are suitable, though the score is slightly lower than the first item. This suggests that the questionnaire is largely well-suited to its intended respondents. A weighted mean of 4.83 indicates that the validators strongly agree that the questionnaire is mindful of cultural differences and is inclusive of various groups.

The weighted mean of 4.67 suggests that the questionnaire is generally considered to be ethically appropriate, with validators agreeing that participant welfare is a key consideration. With a weighted mean of 4.50, the validators strongly agree that the questionnaire is appropriate in terms of its design, language, cultural sensitivity, and ethical considerations. The score indicates that, while the tool is largely seen as appropriate, there may be small areas where minor adjustments could improve its suitability further.

The overall mean of 4.67 further confirms that the questionnaire is highly appropriate for its intended purpose and target respondents. These findings suggest that the instrument is well-designed, culturally sensitive, ethically appropriate, and capable of eliciting accurate and meaningful responses from participants. This high level of appropriateness also indicates that the questionnaire can be confidently used in similar research settings, providing reliable and relevant data for future studies.

The level of validity of the developed questionnaire in terms of relevance was evaluated to determine whether the items are meaningful, significant, and directly related to the objectives of the study. Relevance examines whether each item contributes to measuring the intended construct and addresses the specific difficulties encountered in operating rational numbers. Assessing this aspect ensures that the questionnaire focuses on pertinent content, avoids unnecessary or redundant items, and produces data that are both useful and actionable for the research objectives.

Table 10. *Level of Validity on the Developed Questionnaire in Terms of Relevance*

Items *The questionnaire…*	*Mean*	*Description*
1. assesses all indicators which are relevant to the characteristics and needs of the target population.	5.00	Very High
2. reflects the real-world contexts and challenges identified in existing studies.	4.67	Very High
3. integrates relevant theoretical frameworks from the existing literature on the rational number operations.	4.67	Very High
4. evaluates the consistency with previously validated measures used in related studies.	4.67	Very High
5. obtains an overall impression of the relevance.	4.83	Very High
Overall Mean	4.77	Very High

Table 10 presents the level of validity of the developed questionnaire in terms of relevance. The table assesses how well the

questionnaire aligns with the needs and characteristics of the target population, reflects real-world contexts, integrates relevant theoretical frameworks, and ensures consistency with prior measures. With a perfect weighted mean of 5.00, this indicates that the validators strongly agree that the questionnaire is highly relevant to the participants it is intended for.

The weighted mean of 4.67 suggests that validators strongly agree that the questionnaire does indeed reflect the practical contexts and difficulties identified in existing literature. The 4.67 weighted mean indicates strong agreement among the validators that the questionnaire is theoretically sound and rooted in existing frameworks. With a weighted mean of 4.67, the validators strongly agree that the questionnaire aligns with established measures in the field. A weighted mean of 4.83 indicates strong agreement that the questionnaire is highly relevant.

The overall mean of 4.77 falls in the Strongly Agree category, suggesting that the questionnaire is highly relevant to the research objectives. It aligns well with the characteristics of the target population and reflects both real-world challenges and theoretical frameworks (Brace, 2018). Furthermore, the high relevance rating indicates that the items effectively capture the key dimensions of the construct being measured. This also implies that the questionnaire can generate meaningful and applicable data to support accurate analysis and informed decision-making in the study.

The level of validity of the developed questionnaire in terms of acceptability was evaluated to determine how well the target respondents received the instrument. Acceptability examines whether the items, language, and format of the questionnaire are suitable, understandable, and non-intrusive for participants. Assessing this aspect ensures that respondents are comfortable completing the questionnaire, which promotes honest, thoughtful, and complete responses, thereby enhancing the overall quality and credibility of the data collected.

Table 11. *Level of Validity on the Developed Questionnaire in Terms of Acceptability*

Items *The questionnaire…*	*Mean*	*Description*
1. assesses the clarity of language and instructions.	4.83	Very High
2. evaluates the applicability to the target population and research objectives.	4.67	Very High
3. covers the range of issues related to rational number operations.	5.00	Very High
4. Ensure adherence to ethical standards.	4.83	Very High
5. obtains an overall impression of the acceptability.	5.00	Very High
Overall Mean	4.87	Very High

Table 11 presents the level of validity of the developed questionnaire in terms of acceptability. The table focuses on the clarity of language, applicability, coverage of key issues, adherence to ethical standards, and overall acceptability of the questionnaire. The results demonstrate that the questionnaire is highly acceptable based on these criteria. With a weighted mean of 4.83, this suggests that the questionnaire is very clear in its language and instructions, with validators strongly agreeing that participants will find it easy to comprehend and follow.

The weighted mean of 4.67 indicates strong agreement that the questionnaire is both relevant and suitable for the intended audience. It suggests that the tool has been appropriately tailored to address the needs of the participants while being aligned with the overarching research goals. With a perfect weighted mean of 5.00, this indicates that the questionnaire is highly comprehensive, covering all relevant aspects of the topic. Validators strongly agree that the questionnaire effectively addresses the various challenges and difficulties learners face in rational number operations. With a weighted mean of 4.83, the validators strongly agree that the questionnaire is ethically sound.

The perfect weighted mean of 5.00 reflects the validators' strong agreement that the questionnaire is highly acceptable, in terms of its clarity, relevance, and ethical standards. This suggests that, overall, the questionnaire is considered well-suited for its intended purpose and population, making it a reliable tool for data collection.

The overall mean of 4.87 falls in the Strongly Agree category, which indicates a high level of acceptability. This high score suggests that the validators across all criteria highly regard the questionnaire, making it an excellent tool for the research (Boateng et al., 2018).

Confirmatory Factor Analysis Result of the Difficulties Encountered in Operating the Rational Numbers Questionnaire

Confirmatory factor analysis (CFA) was conducted to validate the factor structure of the Difficulties Encountered in Operating Rational Numbers (DEORN) questionnaire. This analysis determines whether the data fit the proposed model and confirms the underlying constructs measured by the instrument, providing evidence of its validity and reliability.

Table 12. *Confirmatory Factor Analysis Result of the Difficulties Encountered in Operating the Rational Numbers Questionnaire*

Model	*X^2*	*df*	*p-value*	*Remarks*
Baseline model	162.398	15	0.256	Not Significant
Factor model	11.292	9		

Note. The estimator is ML.

Table 12 presents the CFA results of the DEORN-Questionnaire. The Baseline Model value of Chi-Square (X^2) is 162.398, with the Degrees of Freedom (df): 15, while the Factor Model value of Chi-Square (X^2) is 11.292. With the Degrees of Freedom (df): 9 and the p-value: 0.256. The Chi-Square statistic tests the discrepancy between the observed and expected covariance matrices. For the factor model, a lower Chi-Square value (11.292) compared to the baseline model (162.398) suggests a better fit. The degrees of freedom are 9 for the factor model, indicating the number of values in the final calculation that are free to vary. The p-value for the factor model is 0.256, which is above the typical threshold of 0.05. This means the null hypothesis (that the model fits the data well) cannot be rejected, indicating a good fit. The results suggest that the factor model provides a better fit to the data compared to the baseline model.

Table 13. *Additional fit measures of Difficulties Encountered in Operating Rational Numbers Questionnaire*

Index	*Value*
Comparative Fit Index (CFI)	0.984
Tucker-Lewis Index (TLI)	0.974
Bentler-Bonett Non-normed Fit Index (NNFI)	0.974
Bentler-Bonett Normed Fit Index (NFI)	0.930
Parsimony Normed Fit Index (PNFI)	0.558
Bollen's Relative Fit Index (RFI)	0.884
Bollen's Incremental Fit Index (IFI)	0.985
Relative Noncentrality Index (RNI)	0.984

Table 13 shows the additional fit indices to measure the fit of the DEORN questionnaire. CFI compares the fit of a target model to an independent baseline model. A value of 0.984 indicates an excellent fit (values > 0.95 are considered very good). The Tucker-Lewis Index (TLI), also known as the Non-Normed Fit Index (NNFI), the TLI adjusts the fit index for model complexity. A value of 0.974 suggests a very good fit (values > 0.95 are preferred). The Bentler-Bonett Non-normed Fit Index (NNFI), similar to the TLI, this index also adjusts for model complexity. The value of 0.974 again indicates a very good fit. The NFI compares the fit of the model to a null model. A value of 0.930 indicates a good fit (values > 0.90 are considered good). The Parsimony Normed Fit Index (PNFI) takes model parsimony into account. A value of 0.558 is lower than the other indices, suggesting the model may be relatively complex, but PNFI values tend to be lower in general. The Bollen's Relative Fit Index (RFI) compares the chi-square for the model to a null model while adjusting for degrees of freedom. A value of 0.884 is just below the desirable threshold of 0.90, indicating a moderately good fit. The Bollen's Incremental Fit Index (IFI) is similar to CFI but places a greater emphasis on model complexity. A value of 0.985 suggests an excellent fit. The Relative Noncentrality Index (RNI) also compares the fit of the model to a baseline model. A value of 0.984 indicates an excellent fit.

Hence, the confirmatory factor analysis results for the DEORN-Questionnaire indicate that the factor model fits the data well, as evidenced by a low Chi-Square value, appropriate degrees of freedom, and a non-significant p-value. Additional fit indices (CFI, TLI, NNFI, NFI, IFI, RNI) further support the adequacy of the model, all indicating good to excellent fit.

Students' errors in rational number operations reveal misconceptions in their cognitive structures. The DEORN questionnaire may help identify whether learners have reached the formal operational stage, where abstract mathematical reasoning, such as rational operations, becomes possible.

The Difficulties Encountered in Operating Rational Numbers (DEORN) questionnaire is grounded in the Constructivist Learning Theories of Jean Piaget and Lev Vygotsky, which assert that learning is an active process of constructing knowledge through individual cognition and social interaction. Rooted in Piaget's concept of cognitive development, the DEORN helps identify learners' error patterns, conceptual confusions, and misapplications of mathematical concepts, which reflect the current state of their cognitive structures. These diagnostic insights reveal whether learners are still operating within the concrete operational stage or have transitioned toward formal operational thinking, where rational number operations become more abstract and symbolic. Complementing this, Vygotsky's theory highlights the significance of the Zone of Proximal Development (ZPD) and scaffolding, emphasizing that learners' difficulties can be addressed through guided instruction and collaboration. Thus, the DEORN serves as a valuable diagnostic and instructional tool that not only detects the root causes of learners' difficulties in operating rational numbers but also informs teachers' decisions on the appropriate scaffolding strategies to enhance mathematical understanding.

The Difficulties Encountered in Operating Rational Numbers (DEORN) questionnaire serves as a theoretically grounded and practical instrument for identifying learners' difficulties in operating rational numbers. The tool enables educators to design targeted interventions and appropriate strategies that support learners' progression. It not only enhances the assessment of learners' difficulties but also contributes to improving instructional practices and promoting deeper mathematical understanding.

Conclusions

The present study set out to develop and validate the Difficulties Encountered in Operating Rational Numbers (DEORN) questionnaire as a diagnostic tool for identifying learners' challenges in working with rational numbers. Grounded in existing literature and empirical analysis, the study provides a structured means of examining the underlying sources of learners' errors,

moving beyond surface-level performance to uncover deeper cognitive and conceptual issues.

Findings revealed that learners' difficulties in operating rational numbers are multifaceted, encompassing both procedural and conceptual dimensions. Consistent with prior studies, learners struggle particularly with fractions due to persistent misconceptions, incorrect procedures, and the inappropriate transfer of whole number reasoning. These difficulties highlight that errors are not merely computational but are rooted in flawed understandings of number relationships and operations.

The exploratory factor analysis identified four key dimensions of difficulty: misconceptions in performing arithmetic operations, misunderstandings related to numerators and denominators, misapplication of natural number concepts, and errors in fraction operations and interpretation. These dimensions collectively explain the patterns of learners' errors and demonstrate that difficulties are systematic rather than incidental. Such findings emphasize the need for targeted instructional approaches that directly address these domains.

Moreover, the results affirm that the DEORN questionnaire is an effective instrument for diagnosing learners' difficulties across fractions, decimals, and negative numbers. By capturing challenges in both procedural execution and conceptual understanding, the tool enables educators to gain a comprehensive view of learners' mathematical thinking. This reinforces the importance of integrating conceptual clarity with procedural fluency in mathematics instruction.

The psychometric properties of the DEORN questionnaire further establish its robustness. The instrument demonstrated excellent internal consistency, indicating high reliability. In addition, expert validation confirmed that the questionnaire is highly valid in terms of usability, content, appropriateness, relevance, and acceptability. The confirmatory factor analysis also showed that the proposed model fits the data well, providing strong evidence that the instrument accurately represents the constructs it intends to measure.

Given these findings, the study underscores the significance of having a validated diagnostic tool specifically designed to assess difficulties in operating rational numbers. The absence of such targeted instruments has previously limited educators' ability to systematically identify and address learners' misconceptions. The DEORN questionnaire fills this gap by offering a reliable and valid means of informing instructional decisions and intervention strategies.

In light of the conclusions, it is recommended that educational stakeholders utilize the DEORN questionnaire to enhance mathematics teaching and learning. The Department of Education may adopt the tool to guide curriculum improvement and intervention design, while school administrators can use it for pretest and posttest assessments to monitor learners' progress. Mathematics teachers may employ the results to tailor instruction and provide targeted support, and learners themselves may benefit from increased awareness of their specific difficulties. Furthermore, future researchers are encouraged to expand the scope of the study by involving larger and more diverse samples and by exploring intervention strategies based on the identified difficulties, thereby contributing to the continuous improvement of mathematics education.

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Affiliations and Corresponding Information

Kimberly A. Venus
Malapatan National High School
Department of Education – Philippines
venkim1120@gmail.com

Ebrahim Alpe A. Simpal, PhD, LPT
Holy Trinity College of General Santos City – Philippines