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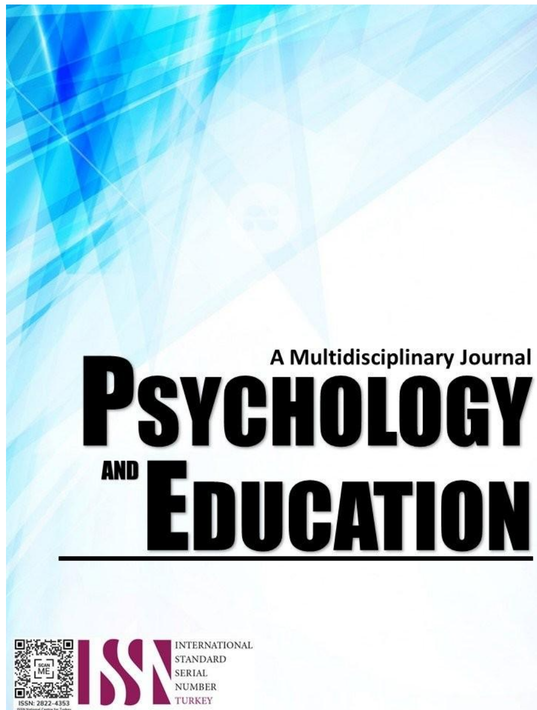

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Research Article

Concrete–Pictorial–Abstract Approach in Developing Students' Understanding of Surface Area of Solids

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Abstract

Learning geometry, particularly the surface area of solids, is challenging for many Grade 8 students due to its abstract nature and reliance on spatial visualization. The Concrete–Pictorial–Abstract (CPA) approach, grounded in Bruner's theory of enactive, iconic, and symbolic representation, has been shown to enhance conceptual understanding in mathematics, yet its application in geometry remains limited. This study investigated the effectiveness of CPA in improving students' understanding of the surface area of solids. Using a single-group pre-experimental design, 30 Grade 8 students participated in a 4-week CPA intervention covering rectangular and triangular prisms and pyramids. A researcher-made 40-item achievement test measured conceptual and procedural knowledge before and after the intervention. Descriptive statistics and paired-sample t-tests were employed to analyze the data. Results indicated that students' mean pre-test score of 52.40 (SD = 8.12) increased to a post-test mean of 78.65 (SD = 6.54), with a statistically significant improvement, $t(29) = 14.32$, $p < 0.05$. These findings suggest that the CPA approach effectively scaffolds learning by guiding students from concrete manipulation to pictorial representation and abstract reasoning, reducing cognitive load and enhancing comprehension. The study concludes that CPA is a practical and evidence-based instructional framework that enhances conceptual understanding, procedural skills, and engagement in geometry, providing educators with a structured method to address persistent challenges in teaching the surface area of solids.

Keywords: *concrete–pictorial–abstract, geometry education, surface area, mathematical understanding, instructional strategy*

Introduction

Learning geometry, particularly the surface area of solids, poses difficulties for many students due to its abstract nature and reliance on spatial visualization skills. Several studies have documented that students often rely on memorizing formulas without fully grasping their conceptual basis, which limits problem-solving ability and long-term retention (Battista, 2007; Clements & Sarama, 2011). When students cannot meaningfully connect geometric properties to their representations, they struggle to apply knowledge flexibly in novel problem contexts (Jones, 2000). Thus, there is a pressing need for instructional approaches that link hands-on experiences to abstract reasoning in order to deepen understanding of geometric concepts.

One such innovation is the Concrete–Pictorial–Abstract (CPA) approach, an instructional framework grounded in Bruner's (1966) theory of enactive, iconic, and symbolic representation. In this model, learning begins with concrete experiences through manipulatives or real-world objects, progresses to pictorial representations such as drawings and diagrams, and culminates with abstract symbols like formulas and algebraic expressions. The CPA sequence reduces cognitive overload and builds stronger conceptual understanding before abstraction (Miller & Hudson, 2007; Purwadi et al., 2019). Studies have consistently highlighted its effectiveness across various domains of mathematics: Moyer-Packenham and Westenskow (2013) emphasized its role in fostering representational fluency, while Carbonneau et al. (2013) demonstrated through meta-analysis that integrating concrete materials enhances mathematical achievement. More recently, Arslan and Ramazan (2018) reported that CPA promotes problem-solving and retention, particularly in arithmetic and algebra.

However, despite the growing body of evidence supporting CPA in arithmetic and algebra, relatively few studies have examined its application in geometry, specifically in developing students' understanding of the surface area of solids. This is a critical gap because geometry requires students to transition from manipulating spatial figures to reasoning with symbolic formulas—exactly the progression CPA is designed to support.

The purpose of this study is to investigate the effectiveness of the CPA approach in improving Grade 8 students' understanding of the surface area of solids. Specifically, it seeks to determine whether instruction that integrates hands-on manipulation, visual representation, and symbolic reasoning leads to higher achievement and more positive attitudes compared to traditional lecture-based teaching. By focusing on an area of mathematics where students frequently struggle, the study aims to contribute evidence on how CPA can bridge the gap between abstract geometry formulas and meaningful conceptual understanding.

Research Questions

This study aimed to investigate the effectiveness of the Concrete–Pictorial–Abstract approach in enhancing the understanding of Grade 8 students of the surface area of solids at a selected public Middle School campus during the 2024-2025 school year. Specifically, it sought to answer the following questions:

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1. What is the students' level of understanding of surface area before and after the CPA intervention?
2. 2. Is there a significant improvement in students' understanding of surface area after exposure to the CPA approach?

Methodology

Research Design

This study employed a single-group pre-experimental design, a type of quantitative research design in which a single group of participants is measured before and after an intervention to determine its effect (Creswell, 2014). In this design, a pre-test–post-test procedure was used to assess changes in participants' knowledge, skills, or attitudes resulting from the intervention. In the present study, all participants were exposed to the Concrete–Pictorial–Abstract (CPA) approach, and their understanding of the surface area of solids, as well as their attitudes toward learning geometry, were measured both before and after the intervention to determine the effectiveness of the CPA instructional method.

Respondents

The study was conducted in a public junior high school during the 2024–2025 academic year. The study took place in a standard Grade 8 mathematics classroom, equipped with essential instructional resources, including geometric manipulatives, diagrams, and visual aids, which supported the implementation of the Concrete–Pictorial–Abstract (CPA) approach. The classroom environment reflected typical teaching conditions, providing a realistic setting for the CPA intervention. This setting allowed students to engage in hands-on activities, create guided pictorial representations, and solve abstract problems, which was crucial for assessing the effectiveness of the CPA framework in improving students' understanding of surface area.

Initially, 42 Grade 8 students were considered for participation, but only 30 students met the inclusion criteria. The participants, aged 13–14 years, were selected from a mathematics class focused on the surface area of solids during the academic year. The inclusion criteria for participants were: (1) enrollment in the Grade 8 mathematics course covering surface area, (2) regular class attendance, and (3) willingness to participate in the study. Students were excluded if they: (1) had prior formal instruction in advanced geometry topics beyond the current curriculum, (2) exhibited irregular attendance during the intervention period, or (3) did not provide parental or personal consent.

To ensure a representative sample, participants were selected based on varying academic performance levels in geometry. This diversity ensured that the sample mirrored the general student population in the classroom, enabling a more comprehensive understanding of the CPA approach's effectiveness across different ability levels.

A purposive sampling technique was used to select participants. This non-probability sampling method involved choosing students based on specific characteristics relevant to the study objectives (Etikan et al., 2016). The intact Grade 8 class, scheduled for lessons on surface area, was purposefully selected to ensure a group with similar instructional exposure and alignment with the curriculum. This approach also facilitated maintaining class cohesion throughout the intervention, ensuring smooth implementation of the CPA method while targeting students who met the study's inclusion criteria.

Instrument

The study employed a researcher-designed achievement test comprising 40 items, including multiple-choice, short-answer, and problem-solving questions, designed to assess students' conceptual and procedural understanding of the surface area of solids. The test was validated for content and face validity by three mathematics education experts, who reviewed alignment with the Grade 8 curriculum, clarity, difficulty, and relevance. Feedback from the experts was used to refine the items.

Reliability was established through a pilot test with students not included in the main study. The Cronbach's alpha coefficient was 0.87, indicating high internal consistency and ensuring the instrument produced consistent and dependable measurements.

Procedure

Data collection began with the administration of a pre-test designed to assess students' baseline understanding of the surface area of solids. This test, which included multiple-choice, short-answer, and problem-solving questions, aimed to measure both conceptual and procedural knowledge. The students were given 40 minutes to complete the pre-test, ensuring they had adequate time to answer all questions. Following the pre-test, the Concrete–Pictorial–Abstract (CPA) instructional intervention was implemented over four weeks, comprising six lessons. The lessons followed a sequential progression: first, students engaged in hands-on activities using geometric manipulatives to explore surface area concepts in the Concrete Phase. Next, they transitioned to the Pictorial Phase, where they created diagrams and drawings to visually represent the surface area of solids. Finally, students moved to the Abstract Phase, where they applied abstract problem-solving strategies using geometric formulas and algebraic expressions to calculate the surface area of various solids. After the intervention, a post-test was administered, which mirrored the pre-test in format, allowing for a direct comparison of students' understanding before and after the intervention. The post-test also took 40 minutes for students to complete. All data collection, including the pre-test and post-test, took place during regular class hours, ensuring a controlled and consistent learning environment for all participants.

Data Analysis

Pre-test and post-test scores were analyzed using descriptive statistics, including means and standard deviations, to summarize students' performance before and after the intervention. To determine whether the CPA approach significantly improved understanding of surface area, a paired-sample t-test was conducted, with an alpha level of significance set at 0.05.

Ethical Considerations

This study adhered to ethical principles to ensure the rights, safety, and well-being of participants were protected. Permission was obtained from the school administration, parents, and the students themselves prior to data collection. Participation was voluntary, and students were informed that they could withdraw from the study at any time without any consequences. Confidentiality and anonymity were maintained by assigning codes to participants and securely storing all data. Furthermore, the study ensured that the intervention posed no harm and that classroom activities aligned with standard curriculum practices. Informed consent and assent procedures were followed, reflecting the researchers' commitment to ethical standards in educational research.

Results and Discussion

Students' Understanding of Surface Area

Table 1. *Students' Level of Understanding of Surface Area before and after the CPA Intervention*

Observations	*Mean*	*SD*	*Interpretation*
Pre-test	52.40	8.12	Intermediate
Post-test	78.65	6.54	Advanced

Note. n=30. 1-20 - Beginner; 21-40 - Basic; 41-60 - Intermediate; 61-80 - Advanced; 81-100 - Expert.

The results from Table 1 illustrate an improvement in students' understanding of the surface area of solids after the CPA intervention. Prior to the intervention, the students had a mean score of 52.40 on the pre-test, placing them in the Intermediate category (41–60). This suggests that, although students had a basic understanding of surface area concepts, they encountered challenges with more complex applications and spatial reasoning. The scores suggest that students were familiar with the general concept of surface area but struggled to fully grasp the underlying principles and their application in various contexts.

After the 4-week CPA intervention, the students' mean score increased to 78.65 on the post-test, placing them in the Advanced category (61–80). This change indicates that the CPA approach helped students deepen their comprehension and develop stronger procedural skills related to surface area. The decrease in standard deviation from 8.12 in the pre-test to 6.54 in the post-test also suggests a more consistent level of understanding among students following the intervention. These findings suggest that the CPA method effectively supported students in progressing from a moderate to a higher level of mastery in surface area concepts, likely through its structured, scaffolded learning process.

Table 2. *Improvement in the Students' Understanding of Surface Area after Exposure to the CPA approach*

Observations	*Mean*	*SD*	*t-value*	*df*	*p-value*	*Interpretation*
Pre-test	52.40	8.12	14.32	29	<0.05	Significant
Post-test	78.65	6.54				

Note. n=30. Significant if p<0.05.

Table 2 summarizes the change in students' understanding of the surface area of solids following exposure to the CPA approach. Initially, students had a mean pre-test score of 52.40 (SD = 8.12), reflecting an intermediate level of understanding. This suggests that while students had some grasp of the concept, their ability to apply surface area principles and visualize geometric shapes in three dimensions was limited. After the 4-week intervention, the mean post-test score rose significantly to 78.65 (SD = 6.54), indicating a marked improvement in both conceptual and procedural knowledge. This suggests that the CPA approach, which integrates concrete manipulatives, pictorial representations, and abstract reasoning, effectively enhanced students' ability to understand and solve problems related to surface area.

The paired-sample t-test revealed a statistically significant improvement in students' scores, $t(29) = 14.32$, $p < 0.05$, confirming that the observed increase in achievement was not due to chance. This significant improvement highlights the effectiveness of the CPA approach in facilitating the transition from concrete understanding to abstract problem-solving. By scaffolding students' learning through hands-on activities and visual aids, the CPA framework appears to have reduced cognitive load and deepened students' conceptual understanding of surface area. These results provide strong support for the use of CPA as an instructional strategy in geometry education, as it appears to foster both greater comprehension and more accurate application of geometric principles.

The results of this study demonstrate that the Concrete–Pictorial–Abstract (CPA) approach significantly enhanced students' understanding of the surface area of solids. The increase in mean scores from 52.40 (pre-test) to 78.65 (post-test), confirmed by a paired-sample t-test ($t(29) = 14.32$, $p < 0.05$), indicates a clear improvement in both conceptual and procedural knowledge. These findings suggest that the CPA framework, which guides students from hands-on experiences to visual representations and finally to abstract formulas, effectively scaffolds learning, allowing students to internalize complex geometric concepts.

These findings align with Bruner's (1966) theory of cognitive development, which posits that learners progress through three stages: enactive, iconic, and symbolic. The CPA approach operationalizes this theory by first using concrete manipulatives to develop foundational understanding, then introducing pictorial representations to bridge to abstract reasoning. Moyer-Packenham and Westenskow (2013) emphasized that such manipulatives foster representational fluency, enabling students to visualize and manipulate spatial relationships before moving to symbolic formulas. This stepwise progression reduces cognitive load and strengthens conceptual comprehension, as evidenced by the significant achievement gains observed in this study.

Previous studies support the effectiveness of CPA beyond arithmetic and algebra. Carbonneau et al. (2013) found that concrete-to-abstract instructional sequences improve problem-solving skills and conceptual understanding across mathematics domains. Similarly, Arslan and Ramazan (2018) reported that CPA enhances retention and application of knowledge in geometry and measurement tasks. By engaging students in concrete and pictorial activities, the present study confirms that CPA provides a scaffolded pathway that connects physical interaction with abstract reasoning, addressing the common challenge in geometry where students struggle to understand formulas without visual or hands-on experience.

The implications of these findings for classroom practice are significant. Teachers can enhance students' geometric understanding by integrating manipulatives, guided diagrams, and structured progression toward abstract problem-solving. This approach not only improves achievement but also fosters deeper engagement and confidence in learning geometry. Incorporating CPA strategies can be particularly beneficial for students who struggle with spatial visualization or abstract reasoning, as it provides multiple entry points for understanding complex concepts. Moreover, the approach aligns with current standards for mathematical practices, emphasizing reasoning, problem-solving, and conceptual understanding rather than rote memorization.

Finally, the study contributes to the growing evidence base for CPA in geometry, filling a gap in the literature where most studies focus on arithmetic and algebra. The findings suggest that scaffolded instruction using CPA can be a practical, evidence-based strategy for improving achievement in three-dimensional geometry. Future study could explore its application across other geometric topics, such as volume, nets, and transformations, as well as its long-term effects on retention and higher-order problem-solving skills. By demonstrating measurable gains in students' understanding of surface area, this study reinforces the value of CPA as a practical teaching framework in middle school mathematics.

Conclusions

The study concludes that the Concrete–Pictorial–Abstract (CPA) approach effectively enhances the understanding of Grade 8 students in the surface area of solids. By progressing from hands-on manipulation of geometric shapes to visual representations and ultimately to abstract problem-solving, the CPA approach enables students to develop both conceptual understanding and procedural skills. This study addresses the primary issue of students' limited comprehension of surface area, demonstrating that scaffolded instruction enables learners to visualize three-dimensional shapes, grasp formulas more meaningfully, and apply their knowledge accurately in solving geometric problems. The results suggest that integrating CPA strategies into classroom practice can make complex mathematical concepts more accessible and engaging, promoting deeper learning and retention. Overall, the CPA approach proves to be a practical and effective instructional framework for improving students' achievement in geometry.

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