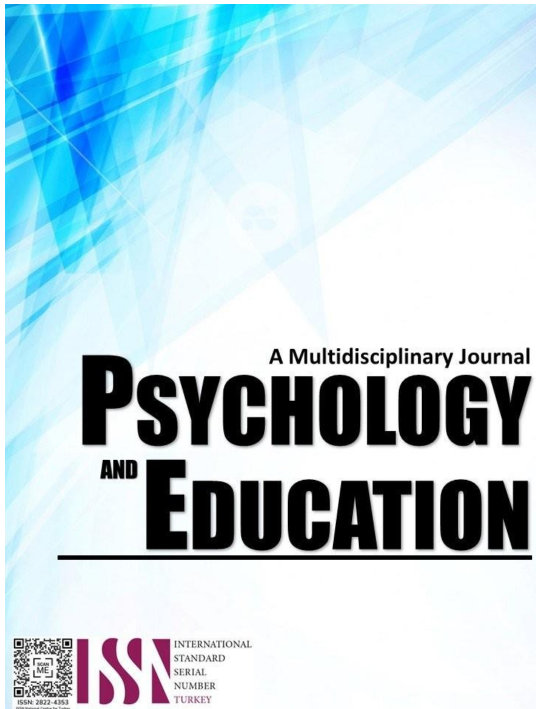
ON HAMMING WEIGHT SEQUENCE



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Research Article

On Hamming Weight Sequence

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Abstract

The study explores the Hamming Weight Sequence (HWS), a fundamental concept in number theory and computing that counts the number of '1's in the binary representation of a natural number. By delving into the properties, generation rules, and theoretical connections of the HWS, the research highlights its mathematical significance and practical utility. The investigation employs descriptive and expository methodologies, systematically analyzing existing literature to establish a robust theoretical framework. Key findings include the formulation of the HWS's recurrence relations and its connections to sequences such as Gould's, Ruler, and Trailing Zero Counting sequences. Moreover, the study underscores the HWS's role in solving Diophantine equations, demonstrating its applicability in mathematical problem-solving contexts. While focused primarily on the HWS, the research contributes to advancing knowledge in discrete mathematics and number theory, paving the way for further studies on its applications across diverse domains.

Keywords: *hamming, weight, sequence, philippines*

Introduction

Mathematics is often seen as daunting, requiring substantial effort and perseverance from those who study it. While its practical applications may not be immediately apparent to everyone, mathematics is undeniably present in our daily lives, from simple household calculations to complex problem-solving in professional settings. The study of number sequences and patterns is an engaging intellectual activity and a vital tool for developing critical thinking and logical reasoning skills. Among the various branches of pure mathematics, Number Theory is a long-standing and expansive field investigating the properties and relationships of integers and natural numbers, providing a rich ground for uncovering complex numerical relationships.

The discovery of the binary number system by Gottfried Wilhelm Leibniz in the 17th century marks a significant milestone in the history of numerical systems. This revolutionary system, using just two symbols, 0 and 1, has significantly transformed technological advancements, particularly in computing. Unlike the conventional decimal system, which uses ten digits, the binary system's simplicity and efficiency have made it indispensable in modern global connectivity and data processing. At the core of the binary system lies the Hamming Weight Sequence, named after the American mathematician Richard Wesley Hamming, who introduced the concept of Hamming codes. This sequence, which counts the number of '1s in the binary representation of a decimal number, provides essential insights into information theory and computer science.

This research offers a comprehensive analysis of the properties of the Hamming Weight Sequence, providing detailed illustrations to foster a deeper understanding of its significance. Additionally, the study aims to explore the intricate relationships between the Hamming Weight Sequence and other notable numerical sequences, particularly those related to binary representations of numbers. Emphasizing the profound implications of the Hamming Weight Sequence, this research highlights its applications in determining the number of solutions to Diophantine Equations, demonstrating its critical role in mathematical problem-solving. Through an insightful exploration of historical contexts, this study provides a thorough overview of the development of the binary number system, tracing its evolution and modern applications, thereby shedding light on the profound impact of the binary number system and the Hamming Weight Sequence on contemporary mathematical thought and technological advancements.

Research Questions

The study delves deeply into the Hamming Weight Sequence's rules, recurrence, and generation. It aims to formulate comprehensive theorems that define its fundamental properties and advance knowledge in pure mathematics. Focusing intensely on this specific topic, the research seeks to establish a robust understanding of the sequence's generator and properties, setting a solid groundwork for future inquiries.

However, the study's exclusive concentration on the Hamming Weight Sequence limits the exploration of related mathematical concepts and potential applications. While it establishes thorough theorems, the scope remains narrow, primarily addressing the sequence's rules and properties. The study also explores theoretical connections with sequences like Gould's, Ruler, and Viabin numbers, yet practical applications and empirical validations still need to be explored. Nevertheless, this research aims to provide a rigorous analysis, laying the foundation for future studies and applications in fields such as Diophantine Equations and broader number theory.

The main objective of this study was to explore the concepts of binary system through the Hamming Weight Sequence. Specifically, the research aims to answer the following questions:

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1. What is Hamming Weight Sequence?
2. What are the properties of Hamming Weight Sequence?
3. How is Hamming Weight Sequence related to:
   3.1. gould's sequence;
   3.2. ruler sequence; and
   3.3. viabin number?
4. How is Hamming Weight Sequence applied to the number of solutions of the Diophantine Equation?

Literature Review

This study is based on several studies about Hamming Weight Sequence, which is relevant to the main topic of this study.

Adams-Watters and Ruskey (2009) highlighted that numeration systems are a fertile ground for generating intriguing integer sequences, particularly through various digit counting statistics. One notable example is the digital sum. For a given number n represented in binary as n=bdbd-1b1b02, the binary digital sum, denoted as s2n, is calculated by summing the binary digits: b0+b1+…+bd. This digital sum is also known by several other names, including sideways sum, sideways addition, population count, and Hamming weight.

Dilcher and Ericsen (2015) introduced an infinite class of polynomial sequence atn;z with an integer parameter t>1, which simplify to the well-known Stern (diatomic) sequence when z=1 and become (0,1)-polynomials when t>2. Leveraging these polynomial sequences, they provided two distinct characterizations of all hyperbinary expansions of an integer n>1. Additionally, they examined the polynomials atn;z independently, deriving a generating function and exploring its implications. Their research also delved into the structure of these sequences, presenting expressions for the degrees of the polynomials.

F. Tichy (1994) discussed how arithmetic functions associated with number representation systems display various periodicity phenomena. For example, Delange's theorem connects the total number of ones in the binary representations of the first n integers to a periodic fractal function. Tichy demonstrated that these periodicity phenomena can be systematically analyzed using classical tools from analytic number theory, specifically the Mellin-Perron formulae. This method naturally leads to the Fourier series involved in the expansions of various digital sums related to number representation systems.

Allouche (2021) highlighted a fascinating discovery made upon reviewing a 1970 paper by R. L. Graham on cube-numbering and its generalizations. In this paper, Graham presented a proof involving an inequality related to the summatory function of the sum of binary digits of integers, offering an elegant and somewhat unexpected solution. Allouche proposed a more straightforward and conventional proof of this inequality and raised questions about potential generalizations. The study of cube-numbering and its extensions involves a peculiar inequality for the summatory function of binary digit sums. Specifically, let ω(k) denote the number of 1's in the binary expansion of the integer k, and define Wn≔0≤k≤nω(k). Then, for all n1, n2 with 0≤n1n2, the inequality Wn1-1+Wn2-1+n1<Wn1+n2-1+1 holds.

McIlroy (1974) studied the Closed formulas provide tight bounds for G(n), the total number of 1's in the binary representations of integers less than n. This function satisfies an extremal recurrence, which gives them the maximum cost of a process that creates a set of n objects by repeatedly merging pairs of smaller sets, starting from n singletons, incurring a cost equal to the size of the smaller set at each merger:

Gn= 1≦i≦n/2maxi+Gi+Gn-i,

where G(1) = 0. The set of pairs (i,n - i) at which the maximum

is attained has an interesting structure.

Allouche et.al (2005), discussed the summation of certain series defined by counting blocks of digits in the B-ary expansion of an integer. For example, if s2(n) denotes the sum of the base-2 digits of n, They show that where + = is the euler constant, - = log4 is the "alternating Euler constant", and N1(n) (respectively N0(n) is the number 0f 1's (respectively 0's) in the binary expansion of the integer n.

The series for + = is equivalent to Vacca's. The formulas for shown in particular that

n≥1s2(n)2n(2n+1)=γ+log4 2

where s2(n) is the sum of the binary digits of the integer n.

Methodology

This study relies on a comprehensive analysis of data derived from published academic articles and literature, serving as a foundational basis to support the exploration of the Hamming Weight Sequence. Using a mathematical research approach, the study combines descriptive and expository methods to facilitate a comprehensive understanding of the intricate properties and theorems associated with the Hamming Weight Sequence. The descriptive method is employed to collect, organize, and analyze the Hamming Weight Sequence information, enabling a systematic presentation of the underlying concepts and principles. Through a meticulous examination of relevant literature, the study offers a comprehensive overview of the foundational aspects of the Hamming Weight Sequence, thereby providing a robust framework for subsequent analysis and interpretation.

Moreover, the study adopts an expository approach to present a detailed and lucid exposition of the Hamming Weight Sequence, elucidating its fundamental properties and theorems in a clear and accessible manner. By employing the expository method, the study not only aims to explain the intricate nuances of the Hamming Weight Sequence but also endeavors to interpret and clarify its dominant concepts and essential aspects. Integrating both descriptive and expository methods facilitates a comprehensive and nuanced understanding of the complexities of the Hamming Weight Sequence, enabling the research to fulfill its aim of comprehensive exposition and interpretation of this mathematical concept.

Results and Discussion

Hamming Binary Sequence. A Hamming Binary Sequence refers to a sequence of 0's and 1's where the data is often represented in binary form for digital transmission and storage.

Hamming Weight Sequence. The Hamming Weight Sequence, denoted as Hn represents a fundamental mathematical concept that counts the number of occurrences of the digit 1 in the binary representation of a natural number n.

Example 3.1 Find the first five terms of the Hamming Weight Sequence.

Discussion: The first natural number is 1, and its binary representation is 12. The number of '1's in the binary representation is 1. Thus, H(1) = 1.

The second natural number is 2, and its binary representation is 102 . The number of '1's in the binary representation is 1. Thus, H(2) = 1.

The third natural number is 3, and its binary representation is 112 . The number of '1's in the binary representation is 2. Thus, H(3) = 2.

The fourth natural number is 4, and its binary representation is 1002. The number of '1's in the binary representation is 1. Thus, H(4) = 1.

Lastly, the fifth natural number is 5, and its binary representation is 1012. The number of '1's in the binary representation is 2. Thus, H(5) = 2.

Thus, the first five terms of the Hamming Weight Sequence are 1, 1, 2, 1, 2.

Theorem 3.1. The nth term of Hamming Weight Sequence is given by the formula.

Hn=log22n n -2n n &2n n -1.

Proof: Let H(n)=log22n n -2n n &2n n -1, where 2n n =2n!n!n! and & is the exclusive AND. To show that the statement is true we can use the formula of 2n n as x and 2n n -1 as x-1. Then let H(n) the number of ones in the binary representations of the positive integer n

Hn=log2x-x & x-1.

Suppose that x & x-1 ≠0, we can say that the x ∩ x-1 ≠0 by the disjoint property. Since x is in the rightmost bit of bitwise AND-operator then x >x-1. Then we consider the binary representations with bitwise AND-operator x >x & x-1. Since for every x in the binary bitwise AND-operator is -1≤ x & x-1 ≤x. Then now we can show is A=x and B=x & x-1. Using the formula say that,

Hn=log2A-B.

Using the identity of logarithm function with difference insight,

H(n)=log2A+log21-BAHn=log2A1-BA=log2A-ABA=log2A-B,

performing antilogarithm base 2 both sides,

Hn=2log2A-B=A-B.

Then, by substitution

Hn=x-x & x-1.

Since x is a binary representation and x & x-1 is a binary

representation bitwise AND-operator therefore H(n)=2n. ■

Illustration 3.1 Find the 1's Counting Sequence of n=2.

H(2)=log22 2 -2 2 &2 2 -1=log26-6&5-1=log26-6&4

Using bitwise operator AND

H(2)=log26-4=log22=1.

Therefore, the number of 1's of n=2 is 1.

Illustration 3.2 Find the Hamming Weight Sequence of n=3.

H(3)=log26 3 -6 3 &6 3 -1=log220-20&20-1=log220-20&19.

Using bitwise operator AND

H(3)=log221-16=log24=2.

Therefore, the number of 1's of n=3 is 2.

On Hamming Weight Sequenc

The Hamming Weight Sequence, denoted as Hn represents a fundamental mathematical concept that counts the number of occurrences of the digit '1' in the binary representation of a natural number n.

The nth term of Hamming Weight Sequence is given by the formula:

Hn=log22n n -2n n &2n n -1.

On Properties of Hamming Weight Sequence

The nth term of Hamming Weight sequence is given by the recursive formula.

Hn={Hn-2⌊log2n⌋+1 Hk if n=k·2i,

where H0=0.

Hamming Weight Sequence can be generated using the piecewise function:

Hn={0, n=0 H⌊n2⌋, n is even H⌊n2⌋+1, n is odd .

It implies that for all n > 0, if n is even, it can be reduced to H⌊n2⌋ , and if n is odd, it can be reduced to H⌊n2⌋+1.

For any non-negative integer k, if n=2k, then the Hamming Weight Hn equals 1.

For any non-negative integers a,b, and p, and ,if n=a·2p+b, then Hn=Ha+Hb.

Let n be a non-negative integer, if Hn is a Hamming Weight of n, then Hn=Hn mod 2+H⌊n2⌋.

On Relationships of Hamming Weight Sequence

Hamming Weight Sequence and Gould’s Sequence. Let Gn be the nth term of Gould’s Sequence. Then the logarithm of the Gn to the base 2 is equal to Hamming Weight Sequence. Symbolically,

Gn= Hn.

Hamming Weight Sequence and Ruler Number. Then the nth term of Hamming Weight Sequence is equal to the predecessor plus 2 minus the Ruler Number. Symbolically:

Hn=Hn-1+2-Rn

where Ruler sequence is denoted by Rn.

Hamming Weight Sequence and Trailing Zero’s in the Binary Representation. The nth term of the Hamming Weight sequence is determined by adding 1 to the value of the (n-1) th term of the Hamming Weight Sequence, and subsequently subtracting the value attributed to Trailing 0's in the binary representation. This relationship underscores the dynamic progression of each term in the sequence, incorporating both the incrementation by 1 and the impact of trailing zeros in the binary representation, when Hn= 0

Tn0=Hn-1+1- Hn,

where n>0.

Hamming Weight Sequence and Binary Length and the number of zero’s in Binary Representation. The nth term of the 0's Counting sequence is precisely defined as the difference between the Length of the binary representation and the Weight of the binary representation. This formulation succinctly captures the essence of how each term in the sequence is determined by considering the interplay between the length and weight of the corresponding binary representation. Symbolically, Hn= LPn -Nn0 .

On Application of Hamming Weight Sequence in the Solution to Diophantine Equation

The Formula of Diophantine in the form of

2mk+2(m-1)+i

is the nth number of the binary 1's representation.

n=2mk+2(m-1)+i.

When the number of m,k,i of the Diophantine equation are $m \geq 1, k \geq 0$ and $0 \leq i < 2(m-1)$ then we can find the nth number of the Binary 1's representation as the number of solutions.

Conclusions

In conclusion, the study highlights the critical importance of the Hamming Weight Sequence Hn in mathematics, particularly for its role in counting '1's in binary representations of natural numbers. The Hamming Weight Sequence its properties established through discrete structures and number theory underscore its foundational significance. Moreover, the sequence demonstrates significant connections with other mathematical sequences such as Gould's sequence, ruler sequence, and Zero's counting sequence. Furthermore, its practical utility in solving Diophantine equations involving binary '1's further exemplifies its relevance in mathematical problem-solving. Overall, the study underscores the Hamming Weight Sequence's pivotal role in mathematical theory and applications.

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